D2 S14 UK

1. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker.

Worker A cannot do task 4 and worker B cannot do task 2.

The amount, in pounds, that each worker would earn if assigned to the tasks, is shown in the table below.

	1	2	3	4
A	19	16	23	–
B	24	–	30	23
C	18	17	25	18
D	24	24	26	24

Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total earnings. You must make your method clear and show the table after each stage.

(Total 10 marks)

max so subtract each from 30

11	14	7	-
6	-	0	7
12	13	5	12
6	6	4	6

⇒

11	14	7	30
6	30	0	7
12	13	5	12
6	6	4	6

RR

4	7	0	23	-7
6	30	0	7	x
7	8	0	7	-5
2	2	0	2	-4

RC

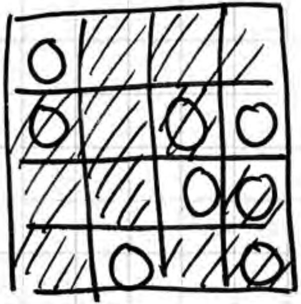
2	5	0	21
4	28	0	5
5	6	0	5
0	0	0	0
-2	-2		-2

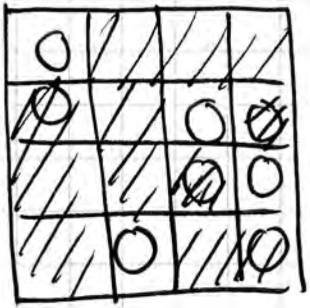
0	3	0	19
2	26	0	3
3	4	0	3
0	0	2	0

0	3	2	19
0	24	0	1
1	2	0	1
0	0	4	0

0	2	2	18
0	23	0	0
1	1	0	0
1	0	5	0

4 lines needed
∴ optimal





A - 1	or	A - 1
B - 4		B - 3
C - 3		C - 4
D - 2		D - 2

Max total = £91

2. The table shows the least times, in seconds, that it takes a robot to travel between six points in an automated warehouse. These six points are an entrance, A, and five storage bins, B, C, D, E and F. The robot will start at A, visit each bin, and return to A. The total time taken for the robot's route is to be minimised.

	A	B	C	D	E	F
A	–	90	130	85	35	125
B	90	–	80	100	83	88
C	130	80	–	108	106	105
D	85	100	108	–	110	88
E	35	83	106	110	–	75
F	125	88	105	88	75	–

(a) Show that there are two nearest neighbour routes that start from A. You must make the routes and their lengths clear.

(4)

(b) Starting by deleting F, and all of its arcs, find a lower bound for the time taken for the robot's route.

(3)

(c) Use your results to write down the smallest interval which you are confident contains the optimal time for the robot's route.

(3)

(Total 10 marks)

	1	4	5	6	2	3
	A	B	C	D	E	F
A	–	90	130	85	35	125
B	90	–	80	100	83	88
C	130	80	–	108	106	105
D	85	100	108	–	110	88
E	35	83	106	110	–	75
F	125	88	105	88	75	–

A-E-F-B-C-D-A

471

	1	5	6	4	2	3
	A	B	C	D	E	F
A	–	90	130	85	35	125
B	90	–	80	100	83	88
C	130	80	–	108	106	105
D	85	100	108	–	110	88
E	35	83	106	110	–	75
F	125	88	105	88	75	–

A-E-F-D-B-C-A

508

'better' upper bound = 471

b)

	A	B	C	D	E	F
A	–	90	130	85	35	125
B	90	–	80	100	83	88
C	130	80	–	108	106	105
D	85	100	108	–	110	88
E	35	83	106	110	–	75
F	125	88	105	88	75	–

AE
EB
BC
AD

RMST = 283

+ DF + EF = 446

'tour nor possible'

c) $446 <$ optimal time ≤ 471

3. The tableau below is the initial tableau for a three-variable linear programming problem in x, y and z. The objective is to maximise the profit, P.

Basic Variable	x	y	z	r	s	t	Value	
r	5	3	$-\frac{1}{2}$	1	0	0	2500	$\theta = 833.\dot{3}$
s	3	2	1	0	1	0	1650	$\theta = 825$ *
t	$\frac{1}{2}$	-1	2	0	0	1	800	$\theta = -ve$
P	-40	-50	-35	0	0	0	0	
		*						

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.

(10)

(b) State the final values of the objective function and each variable.

(2)

(Total 12 marks)

a) Increase y

	bv	x	y	z	r	s	t	Value	
$\theta = -ve$	r	$\frac{1}{2}$	0	-2	1	$-\frac{3}{2}$	0	25	R1−3(newR
$\theta = 1650$	y	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	825	R2÷2
$\theta = 650$	t	2	0	$\frac{5}{2}$	0	$\frac{1}{2}$	1	1625	R3+(newR2
	P	35	0	-10	0	25	0	41250	R4+50(newR2

Increase z

bv	x	y	z	r	s	t	Value	
r	$\frac{21}{10}$	0	0	1	$-\frac{11}{10}$	$\frac{4}{5}$	1325	R1+2(newR3)
y	$\frac{11}{10}$	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	500	R2$-\frac{1}{2}$(newR3)
z	$\frac{4}{5}$	0	1	0	$\frac{1}{5}$	$\frac{2}{5}$	650	R3$\times\frac{2}{5}$
P	43	0	0	0	27	4	47750	R4+10(newR3)

b) $x=0$ $y=500$ $z=650$ $r=1325$ $s=0$ $t=0$ $P=47750$

$P = 47750 - 55x - 30s - 4t$

4. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	2	–1	1	–3
A plays 2	–3	2	–2	1

(a) Verify that there is no stable solution to this game.

(2)

(b) Find the best strategy for player A.

(9)

(Total 11 marks)

	B plays 1	B plays 2	B plays 3	B plays 4	Row Min
A plays 1	2	–1	1	–3	−3
A plays 2	–3	2	–2	1	−3
Column max	2	2	1	1	

Row maximin $\neq$ Column minimax

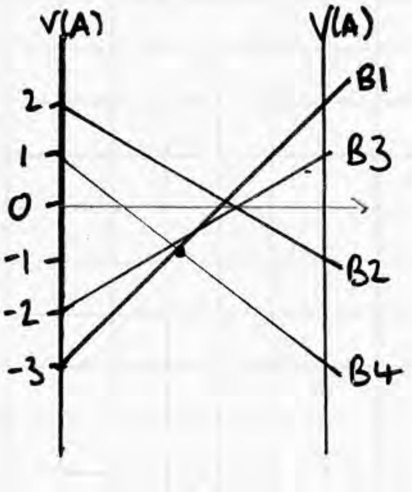
(−3) (1)

∴ no stable solution.

A play 1 prob $= p$
A play 2 prob $= 1-p$

If B play 1 $V(A) = 2p - 3(1-p) = 5p - 3$
B play 2 $V(A) = -p + 2(1-p) = -3p + 2$
B play 3 $V(A) = p - 2(1-p) = 3p - 2$
B plays 3 $V(A) = -3p + 1(1-p) = -4p + 1$

$p=0$	$p=1$
−3	2
2	−1
−2	1
1	−3



$5p - 3 = -4p + 1$
$9p = 4$
$p = \frac{4}{9}$

A should play 1 prob $= \frac{4}{9}$

A should play 2 prob $= \frac{5}{9}$

$V(A) = \frac{20}{9} - 3 = -\frac{7}{9}$

5.

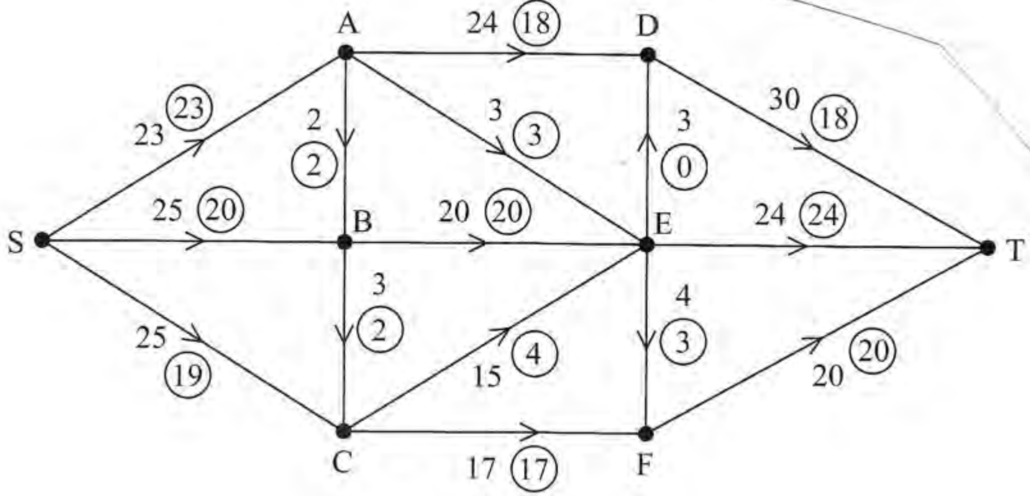


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) Complete the initialisation of the labelling procedure on Diagram 1 in the answer book by entering values along SC, AB, CE, DE and DT.

(2)

(c) Hence use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(d) Draw a maximal flow pattern on Diagram 2 in the answer book.

(2)

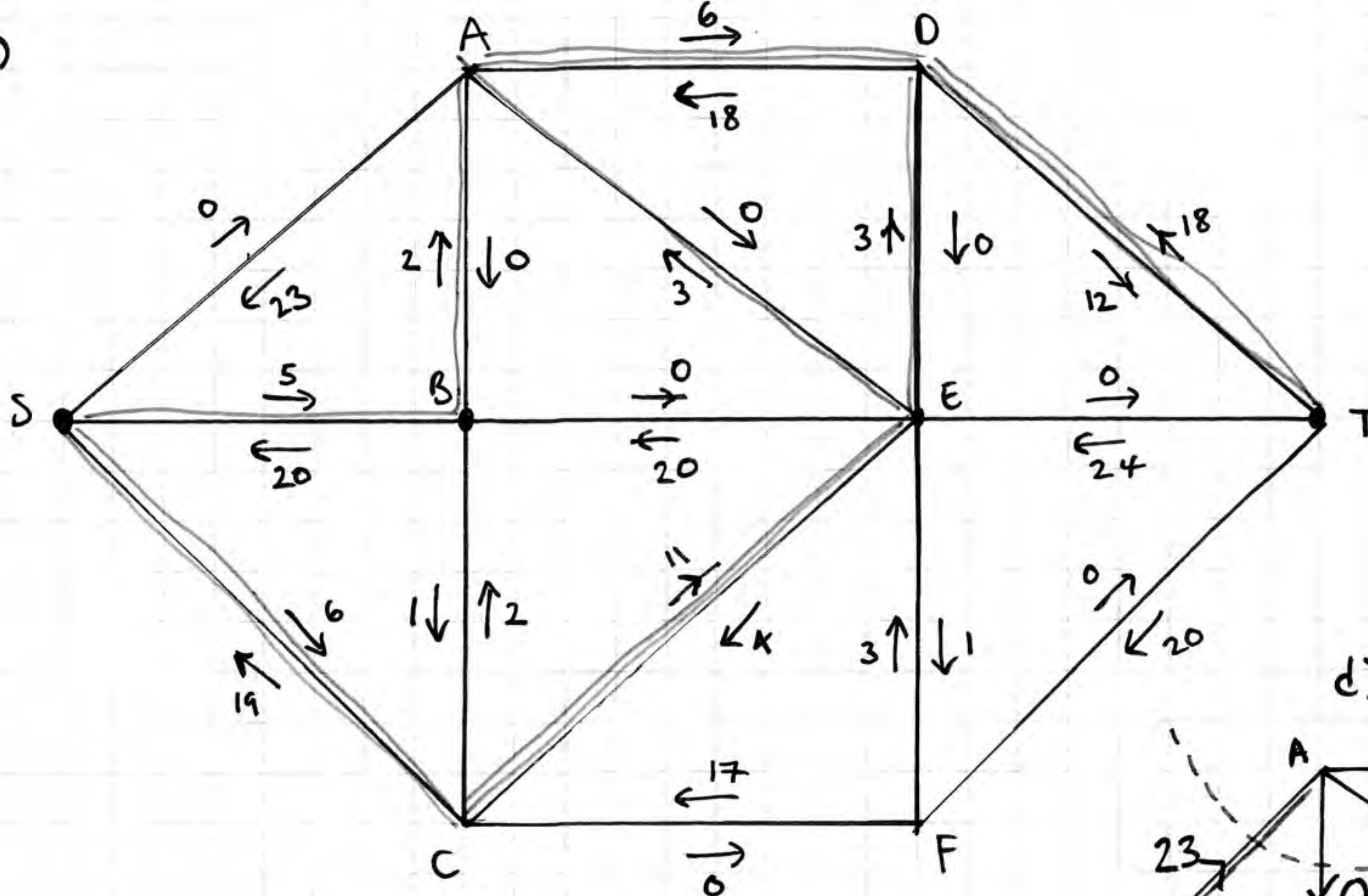
(e) Prove that your flow is maximal.

(2)

(Total 11 marks)

a) Initial flow = 62

b)

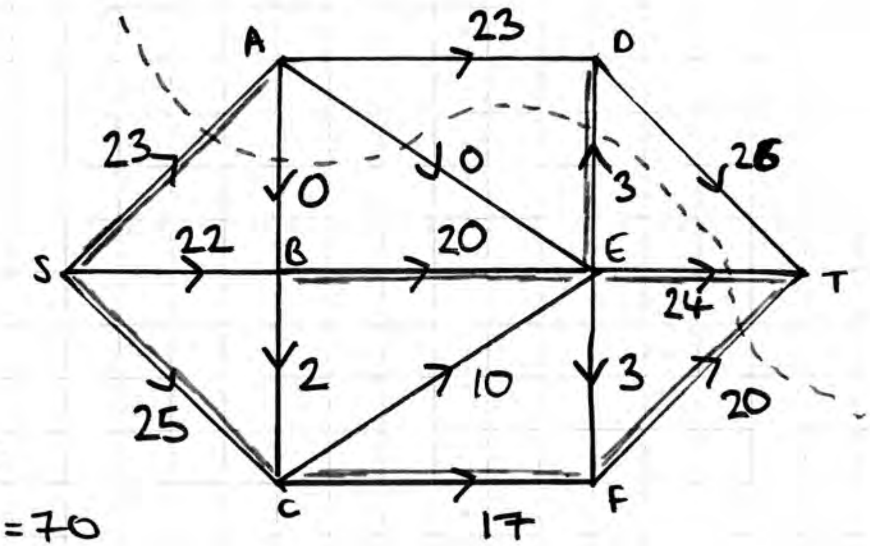


c) SCEDT +3
SCEADT +3
SBADT +2

+8

maxflow = 70

d)



e) Cut through SA, AB, AE, DE, ET, FT only passes through saturated arcs or empty arcs towards source

∴ min cut

∴ by min-cut - max flow theorem

Min Cut = 70

∴ max flow = 70

6. Three warehouses, P, Q and R, supply washing machines to four retailers, A, B, C and D. The table gives the cost, in pounds, of transporting a washing machine from each warehouse to each retailer. It also shows the number of washing machines held at each warehouse and the number of washing machines required by each retailer. The total cost of transportation is to be minimised.

	A	B	C	D	Supply
P	11	22	13	17	25
Q	21	8	19	14	27
R	15	10	9	12	28
Demand	18	16	20	26	

Formulate this transportation problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.
You do not need to solve this problem.

(Total 7 marks)

let X_{ij} = amount transported from warehouse i to retailer j

$i \in \{P, Q, R\}$ $j \in \{A, B, C, D\}$

objective is to minimise transportation costs C in £

$$C = 11X_{PA} + 22X_{PB} + 13X_{PC} + 17X_{PD} + 21X_{QA} + 8X_{QB} + 19X_{QC} + 14X_{QD} + 15X_{RA} + 10X_{RB} + 9X_{RC} + 12X_{RD}$$

Subject to

$\sum X_{Pj} \leq 25$ $\sum X_{iA} \leq 18$

$\sum X_{Qj} \leq 27$ $\sum X_{iB} \leq 16$

$\sum X_{Rj} \leq 28$ $\sum X_{iC} \leq 20$

$\sum X_{iD} \leq 26$

$X_{ij} \geq 0$.

7. A company assembles microlight aircraft.
They can assemble up to four aircraft in any one month, but if they assemble more than three they will have to hire additional space at a cost of £1000 per month.
They can store up to two aircraft at a cost of £500 each per month.
The overhead costs are £2000 in any month in which work is done.

Aircraft are delivered at the end of each month. There are no aircraft in stock at the beginning of March and there should be none in stock at the end of July.
The order book for aircraft is

Month	March	April	May	June	July
Number ordered	3	4	2	4	3

Use dynamic programming to determine the production schedule which minimises the costs. Show your working in the table provided in the answer book and state the minimum production cost.

(Total 14 marks)

Stage	State (in storage)	Action (how many made)	Destination (how many put into storage)	Value (Storage + Costs + Previous)
1 July	0	3	0	0 + 2000 = 2000 *
	1	2	0	500 + 2000 = 2500 *
	2	1	0	1000 + 2000 = 3000 *
2 June	0	4	0	0 + 3000 + 2000 = 5000 *
	1	4	1	500 + 3000 + 2500 = 6000
		3	0	500 + 2000 + 2000 = 4500 *
	2	4	2	1000 + 3000 + 3000 = 7000
		3	1	1000 + 2000 + 2500 = 5500
		2	0	1000 + 2000 + 2000 = 5000 *
3 May	0	4	2	0 + 3000 + 5000 = 8000
		3	1	0 + 2000 + 4500 = 6500 *
		2	0	0 + 2000 + 5000 = 7000
	1	3	2	500 + 2000 + 5000 = 7500
		2	1	500 + 2000 + 4500 = 7000 *
		1	0	500 + 2000 + 5000 = 7500
4 April	0	4	0	0 + 3000 + 6500 = 9500 *
	1	4	1	500 + 3000 + 7000 = 10500
		3	0	500 + 2000 + 6500 = 9000 *
5 March	0	4	1	0 + 3000 + 9000 = 12000
		3	0	0 + 2000 + 9500 = 11500 *

Min Cost = £11500

March	April	May	June	July
3	4	3	3	3